

# Determination of $\alpha_s$ and the Nucleon Spin Decomposition using Recent Polarized Structure Function Data

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## Abstract

New data on polarized  $\mu$ - $p$  and  $e$ - $p$  scattering permit a first determination of  $\alpha_s$  using the Bjorken sum rule, as well as higher precision in determining the nucleon spin decomposition. Using perturbative QCD calculations to  $\mathcal{O}((\alpha_s/\pi)^4)$  for the non-singlet combination of structure functions, we find  $\alpha_s(2.5 \text{ GeV}^2) = 0.375^{+0.062}_{-0.081}$ , corresponding to  $\alpha_s(M_Z^2) = 0.122^{+0.005}_{-0.009}$ , and using calculations to  $\mathcal{O}((\alpha_s/\pi)^3)$  for the singlet combination we find  $\Delta u = 0.83 \pm 0.03$ ,  $\Delta d = -0.43 \pm 0.03$ ,  $\Delta s = -0.10 \pm 0.03$ ,  $\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 0.31 \pm 0.07$ , at a renormalization scale  $Q^2 = 10 \text{ GeV}^2$ . Perturbative QCD corrections play an essential role in reconciling the interpretations of data taken using different targets. We discuss higher-twist uncertainties in these determinations. The  $\Delta q$  determinations are used to update predictions for the couplings of massive Cold Dark Matter particles and axions to nucleons.

When Bjorken first derived his sum rule for polarized deep-inelastic scattering [1], he doubted whether it could be tested experimentally. Several years later, however, he changed his mind [2] and an experimental programme on polarized electron-proton scattering was launched at SLAC [3,4,5]. In view of this, a sum rule for the polarized proton and neutron structure function  $g_1^p$  was proposed [6], based on the dynamical hypothesis that strange quarks and antiquarks in a polarized nucleon would have no net polarization. However, this sum rule was not at all rigorous, in contrast to the Bjorken sum rule which is an inescapable prediction of QCD [7,8]. The first round of polarized  $e$ - $p$  experiments that were consistent, within their stated errors, with the hypothesis of no net strange polarization. However, no measurements were made on polarized  $e$ - $n$  scattering, so the crucial Bjorken sum rule went untested.

Several years later, following the advent of naturally-polarized high-energy and -intensity muon beams at CERN, the EMC made a second measurement of  $g_1^p$  [9,10]. This was more precise than the earlier SLAC measurements, and extended to lower values of  $x \equiv |q^2|/2m_p\nu$ . It indicated a disagreement with the polarized-proton sum rule, corresponding to a non-zero and negative net contribution  $\Delta s$  of strange quarks and antiquarks to the spin of the proton, and a small total contribution of the light quarks:  $\Delta u + \Delta d + \Delta s \ll 1$  [11,12]. This result was surprising from the point of view of the most naïve formulation of the constituent quark model, which would suggest  $\Delta u + \Delta d \simeq 1$ , and  $\Delta s = 0$ , and even compared with more sophisticated versions adapted to fit measurements of  $g_A = \Delta u - \Delta d$  and the hyperon axial-current matrix elements'  $F/D$  ratio, which suggested  $\Delta u + \Delta d \simeq 0.6$  and  $\Delta s \simeq 0$ .

This surprise indicated that our theoretical understanding of non-perturbative QCD was incomplete, and stimulated attempts to remedy this defect. For example, it was pointed out [13] that the EMC result could be understood qualitatively within  $SU(3)_f$  topological models of the nucleon, which describe light quarks via collective fields, incorporate the global symmetries of non-perturbative QCD, and predict  $\Delta u + \Delta d + \Delta s \ll 1$ . Alternatively, it was suggested [14,15,16] that the  $U(1)$  axial anomaly might play a key rôle, which would modify the naïve quark model predictions. It was even suggested that the “sacred” Bjorken sum rule might be violated [17].

In view of the interest in checking the EMC polarized  $\mu$ - $p$  results and extending them to test the Bjorken sum rule, extensive experimental programmes are under way at CERN, SLAC and DESY. An important round of results on polarized  $\mu$ - $D$  scattering from CERN [18] and polarized  $^3\text{He}$  scattering from SLAC [19] were published in 1993, permitting for the first time an experimental test of the Bjorken sum rule. The CERN and SLAC data agreed within their errors on the extracted value of  $g_1^n$  and confirmed the validity of the Bjorken sum rule with a precision of about 12% [20], once the then-available higher-order perturbative QCD corrections [21] were taken into account and allowance made for higher-twist corrections [22,23]. These data also indicated  $\Delta s < 0$  and  $\Delta u + \Delta d + \Delta s \ll 1$ .

Recently, two new sets of data with polarized proton targets have been made available [24,25]. In addition, order  $(\alpha_s/\pi)^2$  corrections to the singlet part of structure functions  $g_1^p$  and  $g_1^n$  have recently been calculated [26] and estimates made of higher-order perturbative QCD corrections to the Bjorken and singlet sum rules [27,28]. Therefore, now is an appropriate moment to reassess the precision with which the Bjorken sum rule has been tested, and to extract new estimates of  $\Delta u$ ,  $\Delta d$  and  $\Delta s$ .

We find that the Bjorken sum rule is now verified with a precision of 10%, once all available perturbative corrections are taken into account. As an exercise, we determine  $\alpha_s$  for the first time from the polarization sum rule data, finding  $\alpha_s(2.5 \text{ GeV}^2) = 0.375^{+0.062}_{-0.081}$ , corresponding to  $\alpha_s(M_Z^2) = 0.122^{+0.005}_{-0.009}$ . On the other hand, we find a consistent pattern of violation of the separate sum rules for proton and neutron targets [6], indicating that  $\Delta s < 0$ . Encouraged by the consistency, we extract values of  $\Delta u + \Delta d + \Delta s$  from the different data sets. These are superficially different if the data are analyzed in the naïve parton model, i.e. neglecting perturbative QCD corrections. However, the agreement between different data sets improves systematically as each higher order of perturbative QCD correction is included. Including the  $\mathcal{O}((\alpha_s/\pi)^3)$  calculation [21], together with estimates of  $\mathcal{O}((\alpha_s/\pi)^4)$  effects in the non-singlet channel [27] and the  $\mathcal{O}((\alpha_s/\pi)^2)$  calculation [26], together with estimates of  $\mathcal{O}((\alpha_s/\pi)^3)$  effects in the singlet channel [28], we find

$$\Delta u = 0.83 \pm 0.03, \quad \Delta d = -0.43 \pm 0.03, \quad \Delta s = -0.10 \pm 0.03, \quad \Delta \Sigma = 0.31 \pm 0.07 \quad (1)$$

at a renormalization scale  $Q^2 = 10 \text{ GeV}^2$ , with a global  $\chi^2 = 1.3$  for 5 degrees of freedom. As an example of relevance of these results, we present at the end of this paper an analysis of the implications of these determination for dark matter couplings to matter, including the elastic scattering of supersymmetric relics and axion couplings to nucleons.

Up to now, the prevailing attitude has been to use polarized structure function data to test the Bjorken sum rule, using the most complete theoretical calculations of corrections and a world-average value of  $\alpha_s$ . The conclusion last year was [20] that the EMC/SMC and SLAC E142 data together verified the Bjorken sum rule to within the available precision of 12%. Here we take a different attitude, more akin to that adopted with regard to the Gross-Llewellyn Smith sum rule [30]. There is no convincing theoretical evidence against the validity of the Bjorken sum rule, which is a solid prediction of QCD. Therefore, its validity can be assumed. Instead, one can use the new polarized structure function data to extract a value of  $\alpha_s(Q^2)$ , whose consistency with other measurements is an *a posteriori* check on the validity of the Bjorken sum rule.

As is well known, data at low  $Q^2$  are particularly sensitive to  $\Lambda_{\overline{MS}}$  and hence at a premium in determining  $\alpha_s(M_Z^2)|_{\overline{MS}}$ . As is illustrated by  $\tau$  decays, even a relatively imprecise determination of  $\alpha_s(\text{low } Q^2)_{\overline{MS}}$  (provided higher-order pertur-

bative QCD and nonperturbative uncertainties are understood) extrapolates to a relatively precise determination of  $\alpha_s(M_Z^2)|_{\overline{MS}}$  [31]. The SLAC E142  $^3\text{He}$  (i.e.  $n$ ) data have  $\langle Q^2 \rangle \simeq 2 \text{ GeV}^2$ , and the E143  $p$  data have  $\langle Q^2 \rangle \simeq 3 \text{ GeV}^2$ , and are hence particularly well placed to exploit this lever arm on  $\Lambda_{\overline{MS}}$ . Since they are at higher  $Q^2$ , the new SMC  $p$  data are less useful in this respect, though they do provide important information about  $g_1^p(x, Q^2)$  at small  $x$ , and enter into the determination of the  $\Delta q$ , as we discuss later.

We use the value of  $\Gamma_1^n(Q^2=2 \text{ GeV}^2) = -0.028 \pm 0.006 \pm 0.009$  given in ref. [20] and the value  $\Gamma_1^p(Q^2=3 \text{ GeV}^2) = 0.133 \pm 0.004 \pm 0.012$  given in ref. [25]. We emphasize that these estimates are based on polarization asymmetry  $A_1(x, Q^2)$  data taken at average values of  $Q^2$  that depend on the bin in  $x$ , which have been converted into values of  $g_1^p(x, \langle Q^2 \rangle)$  by assuming that the  $Q^2$  dependence of  $A_1(x, Q^2)$  is insignificant [20,32] and using standard parametrizations of  $F_2(x, Q^2)$  [33] and  $R(x, Q^2)$  [34] to estimate

$$g_1(x, Q^2) = \frac{A_1(x)F_2(x, Q^2)}{2x[1 + R(x, Q^2)]} \quad (2)$$

No data set indicates a significant  $Q^2$  dependence of  $A_1(x, Q^2)$ , and perturbative QCD calculations [35,36,37,38] lead one to expect that the  $Q^2$  dependence can be neglected at the level of precision required here. The estimates of  $g_1^{p,n}(x, Q^2)$  also require assumptions on  $g_2^p(x, Q^2)$  that are borne out by the latest experimental bounds [24], and consistent with the latest theoretical calculations [39]. Based on the above numbers, we use in our subsequent analysis

$$\Gamma_1^p - \Gamma_1^n|_{Q^2=2.5 \text{ GeV}^2} = 0.161 \pm 0.007 \pm 0.015 \quad (3)$$

for the Bjorken integral.

The small- $x$  behaviours of  $g_1^{p,n}$  have recently been discussed [40] in the context of a non-perturbative model of the Pomeron [41], which suggests that  $g_1^{p,n} \simeq \log(1/x)$  at small  $x$ , rather than  $\sim x^\alpha$ ,  $-0.5 \leq \alpha \leq 0$  [42,43]. Although such an effect would alter the estimates of  $\Gamma_1^{p,n}$  that we use, it would cancel out in the difference that appears in the Bjorken sum rule, since this effect would be due to two-gluon exchange. Hence the estimate in equation (3) would be unaltered.

Perturbative QCD corrections to the Bjorken sum rule have been calculated up to  $\mathcal{O}((\alpha_s/\pi)^3)$  [21] and an estimate was made of the  $\mathcal{O}((\alpha_s/\pi)^4)$  coefficient [27]:

$$\begin{aligned} \Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6}|g_A| \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\pi} \right) - 3.5833 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 \right. \\ \left. - 20.2153 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 - \mathcal{O}(130) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^4 + \dots \right] \end{aligned} \quad (4)$$

in the  $\overline{MS}$  prescription for  $N_f = 3$  flavours, as appropriate to  $Q^2 = 2.5 \text{ GeV}^2$ . The value of  $\alpha_s(2.5 \text{ GeV}^2)$  extracted by comparing (3) and (4) depends on the order in QCD perturbation theory which is used. As we see in fig. 1, the extracted value decreases as one progresses from  $\mathcal{O}(\alpha_s/\pi)$  to higher orders, but shows good signs of stabilizing in  $\mathcal{O}((\alpha_s/\pi)^4)$ . We infer from this analysis a value

$$\alpha_s(2.5 \text{ GeV}^2)|_{\overline{MS}, N_f=3} = 0.375^{+0.062}_{-0.081} \quad (5)$$

which corresponds to

$$\alpha_s(M_Z^2)|_{\overline{MS}, N_f=5} = 0.122^{+0.005}_{-0.009} \quad (6)$$

Note that in evaluating (6) we have used the 3-loop renormalization group equations [44], and matched values of  $\alpha_s$  at the flavour thresholds  $Q = m_c, m_b$ , as is appropriate in the  $\overline{MS}$  scheme. The relevant prescription is given in ref. [45]:  $\alpha_{s-}(m_Q^2) = \alpha_{s+}(m_Q^2)[1 + a(\alpha_{s+}(m_Q^2)/\pi) + b(\alpha_{s+}(m_Q^2)/\pi)^2]$ , where  $a = 0$ ,  $b = 7/72$ . This means that the two-loop  $\alpha_{s-}(m_Q^2) = \alpha_{s+}(m_Q^2)$ , but that the three-loop  $\alpha_{s-}(m_Q^2)$  and  $\alpha_{s+}(m_Q^2)$  are slightly different at  $m_Q$ . Figure 3 compares this determination of  $\alpha_s$  with others, as compiled in [46].

We have not yet included higher-twist effects [22] in this analysis, for two main reasons. One is that their coefficients appear to be rather smaller than originally thought [23], and also because the available estimates differ considerably from one another [47,48,49,50,51,52,53]. For our purposes it is sufficient to take a rough estimate,

$$\delta_{HT}(\Gamma_1^p - \Gamma_1^n) \equiv \frac{c_{HT}}{Q^2} \simeq \frac{(-0.02 \pm 0.01) \text{ GeV}^2}{Q^2} \quad (7)$$

Another reason is that it may be double counting [54] to include this as well as the higher-order perturbative QCD contributions that have now been calculated and estimated. Including the estimate (7) in a fit to the data described above, we find

$$\alpha_s(M_Z^2)|_{\overline{MS}, N_f=5, HT} = 0.118^{+0.007}_{-0.014} \quad (8)$$

The difference between this and the value in equation (6) is indicative of the theoretical systematic error.

This Bjorken sum rule determination of  $\alpha_s$  is encouragingly precise, and quite consistent with the other determinations shown in fig. 3, though not yet competitive with the market leaders. This consistency means that the Bjorken sum rule is verified within 10% precision by the SLAC E142 and E143 data alone, as well as being verified to within 12% precision by the SLAC E142 and EMC/SMC data [20]. The higher precision expected from future SLAC data might enable this determination of  $\alpha_s$  to become truly competitive. A good understanding of the radiative corrections will be needed at this level [55]. The optimal evaluation of the Bjorken integral will also require data from low  $x$ , where the SMC data are available and will continue to provide crucial input. Also, the very high precision data from HERMES [56] at

moderate  $x$  will be very useful. The Bjorken sum rule is making the transition from a test of QCD to a tool for evaluating  $\alpha_s$  within QCD.

Having verified that the Bjorken sum rule is well satisfied by the latest data, we consider the sum rules for proton and neutron targets separately. Their theoretical prediction requires an extra assumption on the singlet axial current matrix elements  $\Delta\Sigma(Q^2)$  or, equivalently, the nucleon matrix element  $\Delta s$  of the  $\bar{s}\gamma_\mu\gamma_5 s$  current. Alternatively, one can regard measurements of  $\Gamma_1^{p(n)} \equiv \int_0^1 dx g_1^{p(n)}(x, Q^2)$  as determinations of  $\Delta\Sigma(Q^2)$ , or equivalently  $\Delta s$ , if one assumes the validity of the Bjorken sum rule. This is the attitude taken here. The fact that  $\Gamma_1^p - \Gamma_1^n$  is in good agreement with the Bjorken sum rule means that the different targets yield consistent values of  $\Delta\Sigma$ , as we now show.

We use in our determinations of  $\Delta\Sigma(Q^2)$  and  $\Delta s$  the calculated  $\mathcal{O}((\alpha_s/\pi)^3)$  corrections to the Bjorken sum rule and the estimate [27] of the  $\mathcal{O}((\alpha_s/\pi)^4)$  corrections used above. We also use the recent calculation of the  $\mathcal{O}((\alpha_s/\pi)^2)$  correction [26] to the singlet part of the sum rule and a recent estimate [28] of the  $\mathcal{O}((\alpha_s/\pi)^3)$  correction to it. If one chooses the renormalization scale  $\mu = Q$ , the proton and neutron sum rules including these corrections can be written as

$$\begin{aligned} \int_0^1 dx g_1^{p(n)}(x, Q^2) = & \left( \pm \frac{1}{12} |g_A| + \frac{1}{36} a_8 \right) \times \\ & \times \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\pi} \right) - 3.5833 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.2153 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 - \mathcal{O}(130) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^4 + \dots \right] \\ & + \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\pi} \right) - 1.0959 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - \mathcal{O}(6) \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3 + \dots \right] \frac{1}{9} \Delta\Sigma(Q^2). \end{aligned} \quad (9)$$

It is worth emphasizing that the  $Q^2$  dependence due to QCD in the singlet channel has two sources, namely the anomalous dimension of the singlet axial current, as well as the coefficient function. However, the  $Q^2$  dependence of  $\Delta\Sigma(Q^2)$  is not large from the  $Q^2$  range of the present data upwards. Note that we have not included any higher-twist contributions, but will comment later on their possible effects.

It is interesting to note that, because of their different  $Q^2$ -dependences, it would be possible in principle to separate the  $SU(3)$  octet and singlet contributions ( $a_8$ ,  $\Delta\Sigma$ ) to either  $\Gamma_1^p(Q^2)$  or  $\Gamma_1^n(Q^2)$ , and avoid in this way the use of  $SU(3)$  [57] relations to estimate  $a_8$  on the basis of hyperon  $\beta$ -decay data. Writing the square-bracket perturbative QCD correction factors on the first and last lines of equation (9) as  $f(\alpha_s)$  and  $h(\alpha_s)$ , respectively, for the octet and singlet contributions, one can write

$$\frac{f(\alpha_s)}{36} a_8 + \frac{h(\alpha_s)}{9} \Delta\Sigma(Q^2) = \Gamma_1^p(Q^2) - \frac{f(\alpha_s)}{12} g_A$$

$$\begin{aligned}
\frac{f(\alpha_s)}{36}a_8 + \frac{h(\alpha_s)}{9}\Delta\Sigma(Q^2) &= \Gamma_1^n(Q^2) + \frac{f(\alpha_s)}{12}g_A \\
\frac{f(\alpha_s)}{36}a_8 + \frac{h(\alpha_s)}{9}\Delta\Sigma(Q^2) &= \Gamma_1^d(Q^2)
\end{aligned}
\tag{10}$$

where the right-hand sides are combinations of measurable and theoretically-known quantities. (Strictly speaking,  $\Gamma_1^d$  in eq. (10) is  $(\Gamma_1^p + \Gamma_1^n)/2$ , i.e. it includes nuclear corrections.) In leading order,  $f(\alpha_s) = h(\alpha_s) = 1 - (\alpha_s/\pi)$ , and the different equations (10) are not independent. At NLO, however,  $f(\alpha_s) \neq h(\alpha_s)$ , and then *each* of the equations (10) allows in principle an independent determination of  $\Delta\Sigma$  and  $a_8$ , provided data with sufficiently high precision are available at different values of  $Q^2$ . One can also combine data from experiments with different targets. The current precision of the world data does not permit a meaningful separation of  $a_8$  and  $\Delta\Sigma$  using this technique.

Using the known variation of  $\alpha_s(Q^2)$  and assuming  $\Delta\Sigma \sim 0.3$  in eq. (10), one can estimate the experimental precision required to make this approach practical. For a range of  $Q^2$  between  $Q^2=2 \text{ GeV}^2$  and  $Q^2=15 \text{ GeV}^2$  we estimate that the right-hand side of eq. (10) would have to be known with a precision much better than  $\pm 0.002$ , including both statistical and systematic errors. Currently E143 quotes [25] error values of  $\pm 0.004$  (stat.) and  $\pm 0.012$  (syst.), so the required improvement in precision would have to be very substantial, especially in the systematic error, unless for some reason the systematic error is approximately independent of  $Q^2$ , in which case it would largely cancel out when comparing the r.h.s. of eq. (10) for the various values of  $Q^2$ . Pending future improvements in the experimental precision, for the rest of this paper we assume the usual  $SU(3)$  value  $a_8 = 0.601 \pm 0.038$  [29], and we return now to the discussion of the existing data.

In order to combine all the data available from different targets, we re-express the measurement on Deuteron and  $^3\text{He}$  targets in terms of the corresponding values of  $\Gamma_1^p(Q^2)$ , assuming the Bjorken sum rule and incorporating all the non-singlet perturbative corrections  $f(\alpha_s)$  on the first line of equation (9). The resulting values of  $\Gamma_1^p(Q^2)$  inferred from the E142  $^3\text{He}$  and SMC D data are shown together with the EMC, SMC and preliminary E143  $p$  data in fig. 4. We see *no signs of convergence* towards the naïvely-suggested [6] value holding if  $\Delta s = 0$ . This situation contrasts with what has been found previously with the Gross - Llewellyn Smith and Bjorken sum rules, namely *good agreement* once the available perturbative QCD corrections are included. Also shown in fig. 4, to guide the eye, is the prediction for  $\Gamma_1^p(Q^2)$  that would be obtained if  $\Delta s = -0.10 \pm 0.04$  which is *highly consistent* with the data. The inclusion of all available higher-order perturbative QCD corrections is important for this consistent picture to emerge. We plot in fig. 5 the values of  $\Delta\Sigma$  that would be extracted from each experiment if one restricted one's analysis to include only low orders of perturbative QCD. The values of  $\Delta\Sigma$  extracted using the naïve parton model, i.e.  $\mathcal{O}((\alpha_s/\pi)^0)$ , are in poor agreement, particularly the E142

neutron measurement done on  $^3\text{He}$ . However, the agreement improves significantly as one proceeds to  $\mathcal{O}(\alpha_s/\pi)$ ,  $\mathcal{O}((\alpha_s/\pi)^2)$  and  $\mathcal{O}((\alpha_s/\pi)^3)$ . (This last analysis includes the estimate of the  $\mathcal{O}((\alpha_s/\pi)^4)$  corrections to the Bjorken sum rule [27] and of the  $\mathcal{O}((\alpha_s/\pi)^3)$  correction [28] to the singlet sum rules.) The overall  $\chi^2$  of the global fit decreases systematically as each order of perturbative QCD is included,  $\chi^2 = 12$  (naïve)  $\rightarrow 4.2$  ( $\mathcal{O}(\alpha_s/\pi)$ )  $\rightarrow 2.4$  ( $\mathcal{O}((\alpha_s/\pi)^2)$ )  $\rightarrow 1.6$  ( $\mathcal{O}((\alpha_s/\pi)^3)$ )  $\rightarrow 1.3$  ( $\mathcal{O}((\alpha_s/\pi)^4)$ ). The fact that the  $g_1^n$  determination of  $\Delta\Sigma$  falls in increasing orders of perturbation theory, whilst the determination from  $g_1^p$  rises, is easily understood from the fact that the higher order perturbative QCD corrections to the Bjorken sum rule are much larger than those of the singlet combinations of structure functions. The main perturbative correction to the non-singlet part comes from the isovector term  $\pm g_A$  which reverses its sign between the neutron and the proton. In the deuteron this term is absent and the effect of the remaining  $a_8$  term is small, and therefore the  $g_1^D$  determination does not change much in increasing orders of perturbation theory.

We conclude that a consistent overall picture of  $\Delta\Sigma$  and  $\Delta s$  is emerging, which leads to the values quoted in equation (1) at the beginning of this paper.

The overall consistency of the different determinations of  $\Delta\Sigma$  and  $\Delta s$  that we find is not affected by the possible  $\log(1/x)$  behaviour [40] of the  $g_1^{p,n}$  that we discussed earlier, since this consistency is simply a consequence of the data and the assumed correctness of the Bjorken sum rule. However, such behaviour would alter the determinations of the individual  $\Delta q$  quoted in equation (1). As pointed out in [58], this could shift the experimental values of the integrals  $\Gamma_1^{p,n}$  and hence the inferred values of the  $\Delta q$  by about one standard deviation towards a higher value of  $\Delta\Sigma$  and a lower value of  $\Delta s$ . As also pointed out in [58], these shifts could be much larger if the small- $x$  behaviours of  $g_1^{p,n}$  were even more singular, but this possibility does not seem to be very well motivated theoretically. For the reasons discussed earlier in connection with the Bjorken sum rule, we have not yet included estimates of singlet higher-twist effects

$$\delta_{HT} (\Gamma_1^p + \Gamma_1^n)_{\text{singlet}} \simeq \frac{(-0.02 \pm 0.01) \text{ GeV}^2}{Q^2} \quad (11)$$

in the extraction of the  $\Delta q$ . If we include them in the global fit, we find

$$\Delta u = 0.85 \pm 0.03, \quad \Delta d = -0.41 \pm 0.03, \quad \Delta s = -0.08 \pm 0.03, \quad \Delta\Sigma = 0.37 \pm 0.07 \quad (12)$$

at  $Q^2 = 10 \text{ GeV}^2$ , and we regard the differences between these and the values in equation (1) as indicative of the possible theoretical systematic errors.

We cannot resist commenting that the value of  $\Delta\Sigma$  that we obtain is much closer to the light current-quark model value of zero in the large- $N_c$  and chiral limit [13] than it is to the most naïve quark model of unity, and that finite- $N_c$  [59] and  $m_s \neq 0$  corrections [13],[60] were estimated to be capable of altering  $\Delta\Sigma$  by about 0.3 .



This picture may be reconciled with otherwise highly-successful models based on constituent quarks, if the latter are regarded as effective constructs that contain non-trivial internal chiral and gluonic structure [61,62,63,64,65,66].

Finally, as an application of these results, we discuss their implications for the couplings to nucleons of certain candidates for non-baryonic Dark Matter. We consider first the lightest supersymmetric particle (LSP), which we assume to be a neutralino, i.e., some combination of neutral, non-strongly-interacting spin-1/2 partners of the  $SU(2)$  gauge boson  $W^0$ , the  $U(1)$  gauge boson  $B$ , and the neutral Higgs boson  $H_{1,2}^0$  in the minimal supersymmetric extension of the Standard Model (MSSM). Since the neutralino has spin 1/2, its couplings to nucleons include a spin-dependent part that is related to the contributions  $\Delta q$  of the different quark species to the nucleon spin. Experiments at LEP and elsewhere constrain the parameters of the MSSM in such a way that the LSP cannot be an approximately pure photino or Higgsino. However, it could be an approximately pure  $U(1)$  gaugino  $\tilde{B}$ , in which case its spin-dependent coupling to a proton is proportional to

$$a_p = 17/36\Delta u + 5/36(\Delta d + \Delta s) = (6g_A + 2a_8 + 9\Delta\Sigma)/36 \quad (13)$$

and the analogous coupling  $a_n$  to the neutron is given by a similar formula with  $\Delta u$  and  $\Delta d$  interchanged, in the limit of a small momentum transfer from the LSP to the proton and  $m_{LSP} \ll m_{\tilde{q}}$ . The couplings  $a_{p,n}$  should be evaluated using the value of the  $\Delta\Sigma$  evolved to a renormalization scale of order  $m_{\tilde{q}}$ , which we take for illustration to be 500 GeV. Using the values (and errors) of the  $\Delta q$  given in equation (1), we find

$$a_p = 0.32 \pm 0.017 \quad (14)$$

$$a_n = -0.10 \pm 0.017$$

We note that the error on  $a_p$  is only about 5%, whilst the error on  $a_n$  is about 17%. This implies that one can estimate relatively reliably the spin-dependent coupling of the LSP to odd-even nuclei such as  $^{39}\text{K}$  or  $^{93}\text{Nb}$ , whose spins are carried essentially by protons, whereas the uncertainty is somewhat larger for even-odd nuclei such as  $^{73}\text{Ge}$  or  $^{29}\text{Si}$ , whose spins are carried essentially by neutrons. Predictions based on the naïve quark model [67] for the spin content of the nucleon are very misleading, particularly for the neutron.

Next, we turn to the couplings of the axion to nucleons, which are given by

$$C_{ap} = 2[-2.76\Delta u - 1.13\Delta d + 0.89\Delta s - \cos 2\beta(\Delta u - \Delta d - \Delta s)], \quad (15)$$

$$C_{an} = 2[-2.76\Delta d - 1.13\Delta u + 0.89\Delta s - \cos 2\beta(\Delta d - \Delta u - \Delta s)]$$

for the proton and neutron respectively. In this case, the low-renormalization scale values of the  $\Delta q$  are the relevant ones, and we evaluate them at the scale  $Q = 1$  GeV, which might be appropriate for the core of a neutron star. We find

$$C_{ap} = (-3.9 \pm 0.4) - (2.68 \pm 0.06) \cos 2\beta \quad (16)$$

$$C_{an} = (0.19 \pm 0.4) + (2.35 \pm 0.06) \cos 2\beta$$

which can be compared with the naïve quark model values given in equation (4) of [68]. We see that the axion-proton coupling is relatively well determined, except for large values of  $\tan \beta$ , whilst the axion-neutron coupling is more sensitive at intermediate values of  $\tan \beta$ , to the experimental errors in the determination of the  $\Delta q$ . As was discussed in [68] and references therein, the dominant axion emission process from the core of a neutron star is likely to be axion bremsstrahlung in nucleon-nucleon collisions, which is sensitive to a combination of  $C_{ap}$  and  $C_{an}$ . The relative weights of neutron-neutron, neutron-proton and proton-proton bremsstrahlung processes are given roughly by

$$C_{an}^2 + 0.83(C_{an} + C_{ap})^2 + 0.47C_{ap}^2 \quad (17)$$

One tries to bound the axion decay constant  $f_a$  by constraining the rate of axion emission from the core of the neutron star. Using the numbers given in equation (16), we find a sensitivity

$$\Delta f_a / f_a = 30\% \text{ to } 50\% \quad (18)$$

This source of error in the constraint on  $f_a$  is much less than other sources of error, in the nuclear equation of state, for example. We conclude that our present knowledge of the  $\Delta q$  is adequate for the purpose of bounding  $f_a$ .

We conclude that the available polarized structure function data are in perfect agreement with QCD, once higher-order perturbative corrections are taken into account. These are indeed essential: the naive parton model is not adequate to describe the nucleon as viewed through polarized lenses. The polarized structure function data already yield an interestingly precise value of  $\alpha_s$ , that future rounds of data could render highly competitive with other determinations. The total nucleon spin fraction  $\Delta\Sigma$  carried by quarks is around 30 %, and the strange contribution  $\Delta s < 0$ . The dynamical mechanism that explains these findings remains a puzzle, and the challenge to reconcile them with the naïve constituent quark model persists. More high-precision data would be most welcome, in order to discriminate between the chiral soliton and axial  $U(1)$  anomaly interpretations of the data, to probe the behaviours of  $g_1^{p,n}$  at small  $x$ , to explore the  $Q^2$ -dependence of  $A_1$ , and hopefully to extract higher-twist effects and  $\Delta\Sigma$  from data at different values of  $Q^2$ . Other experiments, for example on elastic  $\nu N$  scattering [43,69,70,71] and using high-energy polarized proton beams to measure gluon polarization directly, may also help to disentangle the nucleon spin decomposition more completely. This is not only a fascinating way of probing non-perturbative QCD, but also has implications in other

areas of physics, for example in searches for Dark Matter particles, as discussed above. *Floreat* nucleon spin physics!

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## Figure Captions:

Fig. 1. Values of  $\alpha_s(Q^2 = 2.5 \text{ GeV}^2)$  extracted from the E142  $\Gamma_1^n$  and E143  $\Gamma_1^p$  measurements using the Bjorken sum rule and increasing orders of QCD perturbation theory. The last point includes the estimate of the 4-th order perturbative coefficient in [27].

Fig. 2. Values of  $\alpha_s(Q^2 = 2.5 \text{ GeV}^2)$  extracted from the E142  $\Gamma_1^n$  and E143  $\Gamma_1^p$  using the Bjorken sum rule and assuming a given non-zero value of the higher twist coefficient  $c_{HT}$  (cf. eq. (7)). Dashed lines denote error bands corresponding to one standard deviation with respect to the central value of  $\alpha_s$ . Vertical dotted lines denote the range of  $c_{HT}$  given in eq. (7).

Fig. 3. The value of  $\alpha_s(Q^2 = 2.5 \text{ GeV}^2)$  from the Bjorken sum rule, using all available perturbative QCD corrections but neglecting possible higher-twist effects, shown together with a compilation of world data on  $\alpha_s(Q)$  as given in ref. [46].

Fig. 4. EMC, SMC and preliminary E143  $p$  data on  $\Gamma_1^p(Q^2)$ , together with  $\Gamma_1^p(Q^2)$  inferred from the E142  $^3\text{He}$  and SMC  $D$  data using the Bjorken sum rule. The upper continuous curve shows the naïvely-suggested [6] value holding if  $\Delta s = 0$ , together with an error band plotted with dotted curves. The lower curve shows the prediction for  $\Gamma_1^p(Q^2)$  that would be obtained if  $\Delta s = -0.10 \pm 0.04$ .

Fig. 5. The values of  $\Delta\Sigma(Q^2=10 \text{ GeV}^2)$  extracted from each experiment plotted as functions of the increasing order of QCD perturbation theory used in obtaining  $\Delta\Sigma$  from the data. The last point includes the estimate of the 4-th order perturbative coefficient in [27].

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